# Problem Solving: A 21st Century Education 

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Math Prize For Girls
November 14, 2009
Before I even start talking about specific opportunities and strategies for developing problem solving skills, I'll talk about why it's very crucial for your children to not simply think that what's going on in the classroom is enough. Unless you're at a very special school or have a very special teacher, what's happening in the classroom will not prepare you for the challenges that you'll see in college and that you'll see in your careers.

I tried a few times to write the perfect two or three sentences to explain what can happen if you just sit back and say, "Ok, I'll just do what the school tells me to do, and I'll be fine. I'll get hundreds on everything." I haven't been able to beat an email that was sent to me by one of my Princeton University classmates when he stumbled on our website. Here's what he wrote:

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## Result of relying on the Standard Curriculum

> "I certainly wish your website and materials existed when I was in high school. I went through junior high and high school without ever missing a question on a math test, and then took [Math] 103 and 104 at Princeton, which was one of the most unpleasant and bewildering experiences of my life and poisoned me on math for years."

-Princeton University alum

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This is a student who came out of his middle school and his high school and they would have very quickly identified him as a success story: "He got hundreds on everything; he learned everything we taught him; we win. We succeeded with him." And, this is what happened to him at Princeton. This was not uncommon. I saw this a lot at Princeton. Students go through, they
get hundreds on everything, they get a five on the AP Calculus exam, they get 800 on the SAT, and then they go off to Princeton and this happens to them. They run into people like me, who didn't go to as good a high school as he did, but I competed. I did a lot of programs like this. This is part of why programs like The Math Prize for Girls are so important. I'll talk a little bit about why this happened to him so this can help you and your children avoid having it happen to them.


I'll talk about three general areas in which the standard curriculum is failing our top students. The three general areas are the depth of the material, the delivery of the material, and the subject coverage.


First, most of you are pretty familiar with the problem with the lack of depth. In the standard curriculum, the problems are just way too easy. Your children will zip through them really quickly. They're not challenged. They never learn how to confront very difficult problems.

Part of the problem also is that they develop a perfectionist streak. How many of your children are perfectionists, and it drives them nuts when they don't get one hundred percent? They have to get over that. We don't want them to get over that by slacking off. We want them
to get over that by being presented with more meaningful challenges, because if you're always getting a hundred percent on everything, you are not learning efficiently enough. You're not learning as fast as you can and you're not learning how to do things you haven't seen before. What happens is just what we saw with my classmate. If the first time they can't do something is college, they get so used to just being able to do everything because they're "smart," that once they can't do something, they figure, "I've hit the wall. I can't do this anymore. I'm quitting." That's another thing that the tyranny of $100 \%$ encourages in students. It encourages them to think, "I can do all this because I am so smart," and once you can't do it, then you're done, while smarter people can go on. That's just not the case.

Last, you can memorize your way through most of the math texts. These bright students are very good pattern matchers. I could stand up here and teach a class in calculus in Latin to most of these students, if I knew Latin, and they would still be passing. They would still be getting fives on the BC Calculus exam because they'd see the weird little symbols I put on the board, they'd figure out the words, and they'd pattern match. Our curriculum encourages that approach to learning. While it may have been a wonderful thing to be able to memorize lots of facts a hundred years ago, we have Google for that now. We can spend our time developing other skills.


When I went to a teachers' conference, the National Council of Teachers of Mathematics, I started looking at some of the other textbooks. The first book I picked up had one full page about the brilliant career of Nolan Ryan. I'm sure Nolan Ryan could teach us a lot about the importance of hard work and dedication to craft. He's not teaching me much math; there wasn't a bit of math on the page. I'm guessing what they're trying to do is trick the kids into being interested in math. You don't have to trick kids into being interested in math. Math is a beautiful subject in and of itself. I don't need Nolan Ryan to tell me this. The other thing is that the kids are smart. They know they're being pandered to. If you're showing them Nolan Ryan, the first thing they'll think is, "Oh, this isn't important, they're showing me Nolan Ryan." They're not stupid. They see lots of fluff and they're going to tune out.

Also, if you use a lot of drill and kill, for students who are really curious and energetic and interested in the subject, doing the same thing over and over again is boring. But, even worse than being boring, it's counterproductive. When you are drilling the same thing over and over into a student, you are programming them. You are making them become a computer and
the problem is that we already have computers, and anything they can do, they can do better than we can. That gap between them and us will only get bigger. If you are setting a student up to be a computer, to compete with computers, you are setting them up to fail, because they can't compete with computers

The last, and probably the most important, is that the lesson structure is backwards. Your typical class is, "I'll show you how to do this trick. Here's the trick. I'll do it again. I'll do it again. You repeat." That's the typical structure of a textbook, "Here's an example. Follow. You do it." That's exactly backwards. That's not how I learned mathematics. I learned math by seeing lots of problems and then figuring out what the lesson should have been. I figured out what the lesson was from working on all these problems and synthesizing it, making the ideas mine, instead of being something to copy. While doing that, I learned how to solve problems I had never seen before and that's the key skill. More important than any single bit of mathematics they'll learn is how to handle a problem that they've never seen before because that's a transferrable skill. We can take this into law, medicine, economics, and any of the sciences and computer science. We can take that anywhere. If I thought I could teach that by teaching Swahili, I would be teaching Swahili. I just happened to pick math because I think math is the best way to teach this skill.


The subject coverage in your standard curriculum, what does it look like? Prealgebra, algebra, algebra, a little geometry, algebra, algebra, calculus. That's "math." I think a lot of you out here have a mathematical background and realize that there's a lot of discrete math completely ignored, that that's the language of computers. As we already discussed, we can't compete with them for programming. We need to know their language. We need to know how to speak to a computer and discrete math is the language of computing. There's a lot of other beautiful, wonderful mathematics out there that isn't "algebra, algebra, calculus."

One of the big failures of our standard curriculum is, "Calculus is the Holy Grail." The students are taught from a very young age that, "If you get to calculus by $8^{\text {th }}$ grade, you're smart, because calculus is hard." Calculus is easy. How many of your students get to calculus and they think, "Hey, there are two ideas, and they're just doing those same two ideas over and over again," and then they've got three more years of this. It's easy, it's not super challenging, and it doesn't give the students a wide view of mathematics. It almost lies to them -- they get to
calculus in $9^{\text {th }}$ grade and they think they're masters of mathematics, and yet they haven't seen ninety percent of it. They've seen this one narrow slice and it's not even a very challenging slice. The AP calculus exam is extremely easy. You take the AP calculus exam and you compare how students do on it to, say, the AMC contests or contests like this. You'll see lots of kids who get fives on the AP calculus exam but can't get anywhere on tests like the one the girls are taking today. But, you don't see that streak going the other way. You don't see people who can crush these really hard problems like the ones in The Math Prize for Girls and can't do the AP calculus. It just doesn't work that way.

The last thing is that problem solving is the thing you do on the third Friday of the month if you have a little extra time. In my world, it's the whole game. This is the whole game: the goal of education is how to solve new problems. It's not the extra thing. It's not the buzzword of the week. It's the whole point.

Unfortunately, this isn't going to change anytime soon. It's not like things are getting better. It's not like people are figuring this out. They may be paying lip service to it, but when you really scratch and go and talk to administrators, this isn't what you hear. They'll say, "Yes, they're very important. They're very important." But, that's not what they're actually doing. We applied for a grant from the Department of Education, Art of Problem Solving did, and here's one of the comments that we received from one of the reviewers:

Math Prize for Girls
NOVEMBER 14, 2009
> 'W bile challenging and improving the mathematical problemsolving skills of high-performing students are surely every-day objectives of those who teach such students, it is not a problem, relatively speaking, of major import in American education."
> -Department of Education Grant Reviewer

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- wwwartofproblemsolving.com

That's a pretty shocking statement. That's right out of the Department of Education. If we sit back and expect these people to help our best students, we'll be waiting for a long time. And, this was the person who gave us the most favorable review. We didn't get the grant but we went ahead with the project anyway. Now I'm kind of glad we didn't get the grant.


What's the solution to the problem? Typically for schools the solution is acceleration, and most of your children are probably accelerated to some degree. For a lot of the gifted community, that's the answer: accelerate, accelerate, accelerate. Forty years ago, that was probably the right answer because it was the only answer. There was no other game in town; there were no other options. But, acceleration alone is not the answer for a couple of reasons. First, it doesn't really solve the problem. What's the problem? Why do you accelerate the first place? Jane's bored in fifth grade math, because it's too slow, it's written for average and below average students, it's repetitive, and it's boring. So what do you do, you move Jane to $8^{\text {th }}$ grade math. And, what happens? It's too slow, it's repetitive, and it's written for average and below average students. All you've done is put her in a room with people who are three years older. You haven't solved the problem. She's looking at a curriculum that is not designed for her. It may not be designed well for anyone, but it's certainly not designed well for her.

## Math Prize for Girls

November 14, 2009

## Why Challenging Problem Solving is Important

-Presents challenges like the ones students
will face in college
-Prepares students for the intellectual
challenges of many different careers

Next, solving hard problems is way more important than using more and more advanced tools. This is what the typical curriculum does -- it takes us through one-step problems with ever more advanced tools. But, they're not seeing problems that are any harder. The problems on the BC calculus exam are no harder, no more in-depth than the problems they saw in prealgebra. They're one step, just with a much more advanced piece of machinery. It's far more important
to learn how to take lots of different simple tools and put them in combination to solve a hard problem. A one-step problem, that you can do with one fancy tool? Computers can do those; these are not hard problems. Amassing a bunch of different high-powered tools you can use to solve simple problems isn't going to get you anywhere. It's far more important to be able to take a smaller set of tools and be able to do a whole bunch of different things with them and to discover new tools and to build those tools yourself than it is to memorize a bunch of one-trick solutions.

I didn't really come to appreciate all of this until I went to college. I went to a high school that was very average. The graduation rate was around 60-65 percent. Maybe ten percent of my class went to four-year colleges. I went to high school in Alabama and almost nobody went to college outside of Alabama, much less north of the Mason-Dixon Line. I had a very average high school. I had lots of free time. I did not see integration in my calculus class; we spent a full year in calculus class and I never saw an integral. That was the kind of math background I had in my classroom. But, I had a teacher who was willing to take me and some of my classmates to math competitions all over Alabama, and all over the Southeast, and exposed me to greater challenges.

When I went to college, I was a little bit worried. I was up against students who had gone to Exeter, or had gone to this magnet school or that magnet school, and had fives on ten different AP exams. I only took two. I was really worried that I would be at a disadvantage when facing those students. Just in the first semester, I very quickly realized, it was the same game: right at the beginning, you work a little ahead, you figure out what the three big ideas are, and you spend the whole semester figuring out how to put those three big ideas together to figure out everything else. Most of my classmates had never gone through this process, through fighting with really hard problems and realizing that amassing 500 facts to map to 500 problems isn't going to work when somebody gives you the $501^{\text {st }}$ problem. It's better to take those 500 facts and distill them to three ideas and then I can go get anything with them.

They really freaked out and that was what we saw in that first quote. I saw a lot of my classmates do this. Some of them fought through it and, two years later, had learned finally all the lessons that I learned back in high school. Even though they had fives on AP exams and went to these really high-powered and wonderful high schools, they were unprepared for Princeton, but I was. College was very easy. I did not work very hard, I got very good grades, and it was entirely because of the training I had in high school, sitting there looking at a Math Olympiad, having three and a half hours for five problems. Who does that in high school, having three and a half hours for five problems and not knowing how to do any of them? This is a common experience for everyone in college; it's not a common experience in high school. You need to learn how to fight through that, overcome the fear, overcome the panic of "Oh my goodness, I don't understand what some of these words mean and I'm expected to solve this problem." Students need to overcome that and learn how to learn on their own. They'll see this in college. Princeton isn't special in this regard. Every top-tier college is like this. Most colleges are like this. They're setting the bar higher. They're training the students to think.

There's another step when you go off into a challenging career. In high school, they're defining them as a success, they're getting a hundred on everything and they're first in their class. That's not the standard they'll be judged by in their lives. Professionally, they'll be judged by international standards. They will not be measured by the best in their school, the best in their state, even the best in their country. The ones who will go after the cutting edge in this or that will have to be among the best in the world. These are the skills that you need to succeed
in that arena -- you need to know how to solve problems that no one has solved before. You can't just drill and kill your way through that. These are the challenges we need to be preparing our best students for and this is what these sorts of competitions do: they confront students with things they've never seen before.

So now that I've preached enough about that (and I probably didn't have to do a whole lot of convincing), some of you will probably have to do some convincing when you go back to your schools and start arguing for some of the things that I'll advocate in the rest of the talk. I'll talk a little bit about strategies for how to develop these skills. Various programs that I've seen have been really effective with some of these kids. I'll start with the youngest kids. People ask me, "What do I do with my elementary school kids?" It's a very common question. "What math should I be giving them? How many minutes a day should I be drilling them? How many problems should I be doing?" There are only two things you really have to worry about your elementary school kids doing. These two are pretty common to every brilliant problem-solver I've met (almost every one, not every single one).


The first is simply reading. When I was a kid, my parents would often say no when I wanted a toy or when I wanted candy. They never said no to a book. Any book I wanted -baseball statistics -- I'm getting that book. There were books all over my house, all sorts of different books, and I learned to love to read. Reading taught me a lot of different things. First of all, it taught me that there's a whole world that's outside my head, which is kind of shocking to a nine year old. It teaches the students how to love ideas and how to take ideas that are outside themselves that they've never seen before, bring them in, and tie them to other things that they think about. You can start that at a very young age. Almost everybody I know that is a brilliant thinker in any field is also a very avid reader.

As I was staying last night with a very good friend of mine, he was pointing out that reading is probably the highest bandwidth way to get information into your brain. Right? You're sitting there on the internet clicking around, and somebody says "you've got to check this out." You click on it, and it's a video. What do you do? You close the browser. Because if it's an article, you can skim that. You can go through and pull out the information really quickly. For a video, you've got to sit there for five minutes. Unless it's funny or clever, you think, "Forget about it. I'll go read something if I want to learn." That's yet another reason for the kids to be reading.

The second one (for the few young kids that are out there, you'll really appreciate the second one) is play. I can't emphasize enough how important it is to simply play. All the great problem-solvers are very playful people. They spend a lot of time playing even now, not just when they were little kids. When I was a little kid, my dad called me Mr. P, and it wasn't perfect, it wasn't punctual. Whenever there was work to do, I was off playing, so I earned the name "Mr. P." It's just critical. You're learning a lot when you're playing. You're creating problems and solving them. That never goes away.


Moving up to middle and high school, I'll talk about two general strategies. One is extracurricular programs and the other is independent study. I'll start with the extracurricular programs.


The first is simply the benefits of forming a math club. You can get students together. I know some of the parents out here have run Math Circles or have run math clubs at their school. In so doing, you are probably providing most of the students in that club with the most important educational experience they're getting in their school, even though it's not happening in the regular classroom. You can offer more challenging mathematics. You don't have to teach to
what's in the textbook, or to the so-called state standard, or teach to this certain segment of the class. You can teach to the most interested, the most challenging (for you) students.

The second is really important. When you're in middle school and high school, who are the heroes? The basketball players, football players, maybe the band, and it's not just the peers that are sending these signals. It's not just that the kids love the cheerleaders or love the football players. Look at what the school does when they make a football team or make a basketball team or make a band. I'm not saying those are bad things, but they're saying, "These kids who are the best at this skill are very important. They're so important, we're going to hire someone to work just with them and to train just them and make them even better." That's fine. We should be doing it for the math students too. Because I'm a student, I'm looking around, "You've got something special for music, you've got something special for the basketball players, something special for the football players. What do I get? I'm in there in the same class with everybody else. I'm pretty good at this. I love this. I would like to be challenged like they are being challenged," and the school is not doing it. The school is sending these signals too, the school, the teachers, the administrators. When you create a math club, you send a signal to the kids, "This is important. We're going to invest resources in this."

Through a math club, you can also establish a culture of excellence. You see this with all of your large, thriving math clubs and math teams. The top students are, I don't want to say revered, but they are socially rewarded. The students will strive to be like their peers. They learn a lot from their peers. This is one of the wonderful things about The Math Prize for Girls. They're bringing all these girls from all over the place. Some of them might be the only ones within a hundred miles that have the same passion for what they're doing that they do. You bring them all together, and they all start to lift each other up. For some of these students, it's awfully lonely. You see beauty somewhere that no one else in the school is even looking, and that's lonely.

And, you don't have the long view. You don't realize that fifteen years from now, you'll see beauty in places where other people are desperately looking. You don't see the job satisfaction lists. You don't see that the world is in a big recession now but companies are firing the geeks last. You just don't see that. It's hard to convince a twelve-year-old of this. It's hard to convince a twelve-year-old that, when you're 22 and you're really smart, you're really cool. The math club helps show what the world is like out there when you're $25,37,49$, that people who can actually get things done with their mind are rewarded by society and that they're desired, that people want them on their side. They don't see that when they're twelve. The math club helps get past that.

Also, obviously, the social interaction: most of my friends now are people I met through math competitions. The person that I'm staying with here in New York was my fiercest rival in high school until we grew up and then became friends. I'm glad to say that I usually beat him, but that's only because I'm a year older than he is. But seriously, by bringing your daughters here, and those of you flew from all over the place (I know a couple of the girls even came from San Diego which is where I came from), you're doing a wonderful thing for your daughter to bring her here to be around these other people. As much as math competitions have given to me professionally -- this is what I do now -- my friends that I've met through these things are the most valuable thing that I've taken from the events like this that I've been involved with.


Now I'll talk about some of these events. I'll start with the youngest level, the Math Olympiads in Elementary and Middle Schools. It's listed as grade 4 through 8. The very top students will probably graduate themselves out of it well before $8^{\text {th }}$ grade and that's fine. You might even start your second or third grader with some of their problems. They have a few books of problems. They call it a contest, but really it's just an opportunity to show students interesting, different problems that aren't the same thing that they've seen over and over again. They particularly like to pick problems that have lots of different solutions to them. I didn't really appreciate the genius to these problems until I met the director of the program. He showed me one of their problems and asked me how to do it, and I said, "Well I'd just do this and this and this." He said, "ok, well imagine you don't know how to do that," and he took away one of my tools. "Ok, well then I'll do that, and that and that." "Ok, now imagine you don't know how to do that either, now what are you going to do?" And he kept taking away tools and I kept finding different ways to do the problems. That's a great problem. They try to find lots of problems that you can do in lots of different ways.

| Math Prize for Girls November 14, 2009 |
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| Contests |
| Mrymicounins |
| www.mathcounts.org |
| -Premier contest for students in grades 6-8 |
| -School, Chapter, State, National levels |
| -Annual training packet of 300 challenging problems |
| -Club program with group activities |
| Richard Ruscyyk <br> - wwwartofproblemsolving.com |

When you move up to middle school, the gold standard has been MATHCOUNTS. One of the things I love about MATHCOUNTS is that I haven't seen any program get students as
excited about doing math as MATHCOUNTS. It brings the students together which is a really wonderful thing. It's great that you're all here, and not taking The Math Prize for Girls at your school, because you bring everyone together and you get to see the top students come up on the stage. "Wow, look at that, look at what I might be able to do some day." That's a wonderful thing.

This runs grades 6 through 8, and once again like MOEMS, some of your students will graduate themselves out of it before they get to $8^{\text {th }}$ grade, and it's important that you let them do that. I've seen a lot of students in MATHCOUNTS who mathematically get beyond MATHCOUNTS before they're old enough to be out of MATHCOUNTS. Sometimes either through self-pressure or parental pressure or teacher pressure, they keep drilling and drilling in MATHCOUNTS until they hate it. So, if you have a child who's doing MATHCOUNTS and it's clearly getting too easy for them, let them move on. It's ok if they don't go to National. The biggest point of MATHCOUNTS is to get them excited and get them started, and to get them to see that there's more than what's in the textbook. Once you've done that, you've won the battle. I have seen few things in middle school do that as well as MATHCOUNTS does.


Once you get up to high school, there are the American Mathematics Competitions. They do have a middle school activity, but most of these are high school programs although a lot of middle school students do participate. These are the series of tests that determine the national team, both for the China Girls Math Olympiad (I understand several of the young women that were on the team this year and last are here at the event today) and the U.S. International Math Olympiad. Working on these problems is probably where I learned the most about how to stare at problems that I have no idea how to do.

The top 60 or so students are invited to the Math Olympiad summer program which was a very important experience for me as a student. I went to the Math Olympiad summer program my sophomore year and I was quite sure that I was the smartest person I knew. The first two years that I went to the Math Olympiad program, I saw a total of around a hundred problems on various practice tests. [Zero,] that's how many I solved -- not a one. It was a pretty sobering experience for me. I learned a lot about where I was mathematically and where I needed to be, so that by my senior year I could actually do a few of the problems. That prepared me for going to Princeton and sitting in front of a physics test, an economics test, an engineering test or any sort of test that has four problems in four hours, and I don't know how to do any of them when I
start. But, I know how to do most of them by the time I'm finished. That's where I learned how to fight through that.


There are lots of other contests. There are a lot more in high school than there are in the earlier grades. Here are a few of my favorites. Some of these are very focused on themes. The ARML was my favorite when I was a student, because it was the first time I was on a whole math team, a 15-person team where everybody on the team was excited about math, and there were several people on the team that were as good or better at it than I was. The USAMTS is one that our foundation runs. This is funded by the National Security Agency. The reason they fund this is that it's more like what they do than any of the other contests because the students are given 30 days or more to work on the problems, they have to write up full solutions, they can use a computer, they can use books, and they can research. The NSA describes this as much more like what they do, like what a mathematician does. You send a paper exam and you have mathematicians and some college students grade everything and give comments back. The Purple Comet is a free competition that's on-line and I believe they have a middle school and a high school level. The Mandelbrot now has a separate, individual line of competing portion.


I'm not the only one that's realized that these contests are an indicator of success in college and beyond. If you take a look at college applications, they're not asking for the National Latin Exam. They're asking for APs and SATs, but you know what those will look like
on the MIT and CalTech applications. They're asking for AMC scores. I know a professor at MIT who said that for the math department, when they're evaluating, they get a $5 \times 7$ index card with the information about the students. You send in a big packet; they get a $5 \times 7$ index card. These scores are on it. They obviously think there's something important going on here. Some other schools give scholarships for top AMC scores.


Math contests aren't the only way you can develop these skills. There are a lot of other ways to get a very broad-based education in mathematics and problem-solving outside the classroom. One of them is Math Circles. These are the cities. Even as this list was going up there are probably a few more starting, so if you live in any of these cities, you can find a Math Circle. (There are a couple of young women from the San Diego Math Circle here today. I'll talk more about the San Diego Math Circle in a little bit.) Math Circles have a lot of different structures. Some are focused on contests, some are focused on higher mathematics, and some are focused on just getting kids together and going in whatever direction that the kids are going and exploring whatever it is they feel like exploring.


I'll move on now to independent study. I would describe most of my high school education as independent study. It wasn't formal independent study. I wasn't off in my own room, separate from the math class, although nowadays with the way things have become with
students being so overloaded with work, that's actually what I'd advocate. I was on the advisory board for the Cogito website, which is a website for very high-performing math and science students. They had little break-out groups when they brought the advisory panel together and I was in a room with a bunch of very successful researchers in math, computer science and robotics. They gave us an hour to come back with recommendations and we weren't really given much focus for what we were supposed to discuss. There was only one thing that we could agree on, that it's very important for students to be given time to explore their academic passions. That was deemed even more important than pairing students up with researchers or giving them a way to accelerate through mathematics or through science. Simply block out a portion of the day and say, "This is yours. You want to go read Martin Gardner books, fine. If you want to play a video game, fine. Do whatever you want, find something you're passionate about, do that and get good at it." Ok, we don't want them playing a video game, but we have to carve out time and let them go and pursue what they want to do and figure out how to attain their goals.

If we keep laying tracks in front of them, what happens when they get to the end? At some point, they have to figure out how to do things for themselves. Looking back now I realize my high school education (again, very average high school; I hardly ever had homework and hardly ever had any real challenge in my classroom) was perfect for me. I'm an entrepreneur now, building a company, and where are the tracks for that? Where are the tracks for developing a math curriculum outside a major publisher? There are no tracks for this. I wrote a couple textbooks while I was in college; there are no tracks for that either. Where did I learn how to do these sorts of things? I learned how to do it in high school. No one told me I was learning this, but I was given lots of time, I had goals, and I had to figure out how to reach them.

That's why I think independent study is so important for these students because wherever they go, that's what they're going to do. They'll go through, they'll get a PhD and become a leading researcher in X . Once they get out to that frontier, there will not be anybody saying, "That way. Do that." They'll have to figure that out. We can start teaching them that way earlier. They don't have to wait until they're 27 to start thinking about these things. This is life. Life is independent study.

For our kids in high school, we plan their day from the minute they wake up until the minute they go to sleep, and the minute they go to sleep is getting later and later and later. If you can break out time during their day and say, "Find your goals and go do what you want," you've done a huge service for your kids. A lot of my students now will do this. They'll go to the school, they'll negotiate with the teacher and find ways to get out of the classroom. They'll have to show that they can do this stuff. They'll have to pass the tests. But, if they can do that and you have a teacher who says, "Look, there's nothing I can do with this student and still serve the other 24 kids in the room," and that teacher will let them go and pursue their own path, that's the best that we can do. For a lot of kids, that's the best that can be done.

Part of this is just taking off the ceiling. When you put the tracks in front of them every step of the way, you're telling them, "Here's the goal." "Now what?" "Here's the goal." You're putting a ceiling on them no matter what you're doing when you're highly structuring what they're doing. You've got to take that away and let the kids explore and go out to their boundaries and push on them rather than creating artificial boundaries in front of them. Also, you're less likely to bore students into losing interest. After doing the same thing over and over again, they'll focus on this little interesting thing over here and work on that for a while, and like I said, become more mature learners. They learn how to learn.

This is something I think about a lot, even now as I'm teaching, to try to get students to understand how to use the internet to learn things. You can learn things now much more efficiently. Look at what's happening in the chess world. I don't know how many of you are involved with chess, but chess has been radically altered by the internet. Students can learn much more efficiently now, and the same thing is true in math. That's because it enables students who want to learn to go out and learn themselves without waiting for the information to come to them. And, as I mentioned, it gives students time to go out and learn about the things they care about most.

It's nice to be well-rounded; I don't want to say that it's a bad thing to be well-rounded. They can go out and play with the basketball, and clearly I like to talk. But, it's a specialists' world now. The world is not looking for someone who's good at 17 things but not great at anything. I'm looking for people who are great at three things. I don't care what those three things are. I want them to find something they're passionate about and get really, really good at it. Then you'll be happy, especially if we can tack some math skills and programming skills on top of that, because then you can do anything.


Summer programs, in a sense, are loosely-guided, if at all guided, independent study, and they also give students a launching-off point for working for the rest of the year. As you can see, there are a bunch of them. The one on the top, separated from the others, is the only one on here that's primarily focused at middle school. The rest are either primarily focused on high school or middle and high school. There are a lot of factors that go into choosing a summer program.

If you are interested in sending your child to one of these programs, one of the factors that you should consider is emphasis. Some of these programs focus very much on math competitions while others focus very much on exposing students to a wide range of areas of mathematics and they don't care about the contests at all. They're focusing on trying to inspire another generation of mathematicians. Some of these do a little bit of both. Some of them are very highly-structured, so, "Problem set, problem set, test; problem set, test." Some of them are not structured at all. "You want to come to class, great; you don't want to come to class, great. You do what you want to do, here's six things you can do every hour." You'll know best and your child will know best which of these suits the child. Do they need the structure? Do they want to do contests? Do they want to study math?

The one on the bottom right, RSI, is something for students who are interested in research. They help students develop projects for research competitions which we'll talk about in a moment. That is a very tough-to-get-into program, it's very prestigious, it's free, and it's for students who have just finished their $11^{\text {th }}$ grade year. A lot of their students go on to the major research competitions. That's something that should be on your radar if you have a child who's $11^{\text {th }}$ grade or earlier and is interested in developing a research project or something along those lines.


These are the various research competitions that are out there. Some of these come with $\$ 50,000$ to $\$ 100,000$ awards. These are a really big deal. To be fair, it's very hard to go it alone on one of these. It is possible -- one of our students, a homeschool student, found mentors on the internet with one of the Davidson fellows, and largely did the project on his own -- but nearly every winner I know of these competitions has worked with a professor of some sort pretty closely. Some of that has often started at RSI. If you are interested in one of these, probably one early step should be building a relationship with your local university, trying to find a mentor.


There are also other Olympiads besides the Math Olympiad and here's a list of them. For the USACO Olympiad, all four students on the American team were Art of Problem Solving students. We don't teach computer science, not yet; we will some day. That's just an example.

Two or three of the Linguistics students were ours as well, and probably one or two of the others. I've taught people who have been on the other teams. That shows that the way to succeeding in all these is to develop problem-solving skills. Most of the students are doing that through mathematics, as kind of a first step to get into these.

Math Prize For Girls
November 14, 2009

## Challenges to Serving the Top Students



You'll see a few challenges in serving these top students and the first is (we've already talked about a lot of this) convincing the schools they have a problem, getting them to understand that, when the student is the smartest person in the room, that student needs to find another room. A lot of schools don't understand this; a person who's getting a hundred is no longer a problem. So, you may have to do some work to explain that.

Closely related to that are equity issues. They'll say, "Well, if we let Mary do that, then we have to let everybody do that." They somehow solve these problems for the football team and for the band. They can solve these for the top math students as well. This is something that you'll have to be ready for. One simple answer for that is, if they allow Mary to do that, they'll have to allow everybody to that, "Ok!" If you have five other students who are at Mary's level in math and at her level of interest in math, and who go off and work through a harder curriculum, you should be thrilled to let them do that, because most of these things are pretty self-selecting.

You know that as parents, you can force them, you can stand there with the hand behind the back, until thirteen, fourteen, twelve. I don't know where it is, but that line's somewhere around middle school. Once you get up around fifteen and sixteen, you will not win that battle. That's something I see with MATHCOUNTS a lot as well. I see parents push their kids all the way up onto the stage at national MATHCOUNTS. Those kids quit. Not all of them, but most of them do. The kids who don't have the parents behind them, with both hands pushing all the way, are just getting started. People probably preach to you all the time about not pushing the kids too much. I've seen a lot of the best kids in the country over the last five or six years, and the ones who are getting pushed the hardest are the most likely to quit. We can't have them quit. We need them ten, fifteen, twenty years from now. We can't have them quit.

You also have to convince the students that there's a problem and that can be sometimes harder. A student's getting 100, 100, 100. The student may not realize that there's a problem, that there's something more out there. I think probably the best strategy for this is to keep exposing the students to interesting things. You don't want to force them too long. I told my

San Diego Math Circle parents that you can force your kids here once or twice. But after that, you've got to ask them. You've got to sit them down and say, "Do you want to keep coming?" If they say "Yes," you've won, and if they say "No," you've got to stop. Maybe come back in a year or two.

You have to keep showing the students that there are more interesting things out there and hope that something catches. Maybe it's robotics, maybe it's programming, maybe it's biology, maybe it's linguistics. Keep showing them interesting things, and if they have a curious, active mind, something will grab them.

The next thing's very important, and that's to get out of the way and to clear obstacles away. One of our roles as educators of these top students is removing barriers.

Next of course, is calculus, calculus, calculus. You've probably had this. You've got a kid who says, "Mom, teach me calculus! I want to learn calculus now!" It comes from the kids too. They're picking it up from their peers and they know that if they're in $7^{\text {th }}$ grade and learning calculus, they'll be gods. It's not just the curriculum that's set up. The kids internalize that. You'll have to battle that, show them other challenges, and show them that there's something more interesting, more challenging out there than precious calculus.


This is my favorite part of the talk, but I've gone on so long with the other stuff, I'll go through this very quickly. We'll talk about some resources for learning math from a problem solving perspective. We quickly talked about the Art of Problem Solving books. I don't know how many of you have your daughters using some of these materials. Basically, we're writing the books we wish we had when we were students.

I got the idea for the structure of these books when I was listening to a talk by a friend of mine who was explaining how he learned math, just the way I did. We learned by doing lots of problems and then assembling all the problems in our head somehow. These books are structured by giving the problems at the beginning of each chapter instead of at the end. So, instead of doing "Lesson, lesson, lesson, copy," we say, "Problems! Figure out how to do them, and then we'll show you how we did them, and you can compare." That way the math becomes their math instead of math that somebody else told them. Even more importantly, they're learning how to tackle new things, which is more important than the things they are actually tackling. This is our series for middle school and early high school and we are continuing that
into high school. We just sent a precalculus book to the printer; I'm very happy about that. The calculus book will go in the next couple of months.


Here are the backgrounds of some of the authors of the books. We're taking all of our experiences from doing this and putting them into the books.


Some of you are probably familiar with the old Art of Problem Solving books. These are the books that Sandor and I wrote when we were finishing college. These are designed specifically for math contests. The other books are a full curriculum designed for highperforming students. We started the Mandelbrot competition at the end of my first year of college and we ran into a little bit of a problem with the competition because the tests were really hard and the scores were pretty low. We had to make a decision as to whether to dumb down the tests or try to teach the kids. We did something that's a little novel in American education -- we opted to teach the kids. That's where this started.
Math Prize for Girls
November 14, 2009

## Resources for a Math Problem Solving Education BOOKS

Mathematical Circles by Fomin, Genkin, Itenberg
Art and Craft of Problem Solving by Paul Zeitz
Problem Solving Strategies by Arthur Engel
Olympiad problem books and preparation texts by Titu Andreescu and/or Zuming Feng
Richard Rusczyk
Richard Rusczyk

- wwwartofproblemsolving.com
- wwwartofproblemsolving.com

We're not the only ones writing books. These are some of the other books that we like. The first book is for younger students, really strong middle school students, early high school students, and that comes from the experience of the Soviet Union, which has a rich culture of mathematics where professors work a lot with the middle school and high school kids. They weren't so good at building economies, but they were very good at building mathematicians. The rest of these books were written by team leaders of various national teams. Three of them, Paul, Titu and Zuming, are past or present coaches of the American IMO team and Arthur Engel was the coach of the German IMO team. These are for students who are in high school and who are preparing for the USAMO.
MATH PRIZE FOR GIRLS
November 14, 2009
Resources for a Math Problem Solving Education
CLASSES
Online classes for problem solving available at:
-Art of Problem Solving
-Belin-Blank
-CTY
-EPGY
-TIP
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anwartofproblemsolving.com

There are also lots of classes on-line. We have classes at Art of Problem Solving. A lot of the other gifted centers have on-line classes, and some of them we know are good because they use our books, so they must be good. There are lots of different classes on line and again they're all structured in different ways. Some of them are very self-paced and don't have much interaction with instructors. Some of them are more highly structured and have more interaction with instructors. Some of them are focused solely on math competitions. Some of them are a full curriculum. Some are focused on acceleration through a regular curriculum (I'm not as fond of those, obviously). There are lots of options there for you to explore.

| Math Prize for Girls November 14, 2009 |
| :---: |
| Resources for a Math Problem Solving Education WEBSITE |
| Www.artofproblemsolving.com |
| -Books |
| -Classes |
| -Articles on problem solving topics |
| - LaTeX tutorial for typesetting mathematical writing -AoPSWiki |
|  |  |
|  |
| Richaral Rusczyk <br> - wwwartofproblemsolving coms |

Our website is kind of the center of the middle and high school math geek universe. I would have spent a lot of time there when I was a kid. Basically, the kinds of things we're building on our website are all sorts of stuff we think we would have wanted when we were thirteen.


This is a game some of your kids probably play with a lot. It's basically like the MATHCOUNTS countdown round. We've got five or ten thousand problems in the database now and the kids can compete against other kids from all over the world.


We're developing now what we'd applied to get a grant for, an adaptive learning system. There are a few other adaptive learning systems out there, but they are specifically designed for remedial math because that's where the money is. This one's designed for your children. It consists of a database of problems and some video lessons. (So, if I haven't completely bored you to tears, you can hear me talk some more on this.) Sitting behind this is an algorithm that watches the students do problems and then picks out future problems based on the past work within the system. We're just getting started with this. We hope to expand it both for younger students and for older students.


The heart of our website is our on-line community, our message board, which I think now has over one and a half million posts. The kids are mostly talking about math and a little bit about video games. They're mostly asking each other questions, challenging each other and learning from each other. We're trying to reach those people who, like me, were in schools where they were pretty alone and wanted somebody to talk to and learn from.


I'll go through a few opportunities in college. It's extremely important for these students, and this ties in with what I mentioned earlier about giving students time. You know what it's like out here, once you get out in the real world. It's hard to find time to explore your interests and to explore your passions. It's rare for people to be able to choose like I did. I was very lucky that I was able to choose, to significantly change careers, more than once. Not everybody has that latitude for new things, because they have responsibilities, they have families or for financial reasons. In college, in high school, students have a lot of time and they should take that time to explore a lot of things.

One of the most important things you can do during summers is use those summers to do lots of different things. Don't just do the same thing every single summer. You have to expose yourself to a lot of different experiences. For one, you may find a thing you really love doing, and that's great, and then you win. But, you can also be marking things off. I spent the summer in a research lab at a university, X. I spent a summer in industry, at DuPont in a chemical engineering lab, X. That helped me a lot because then I went to grad school and I left in 8 weeks. I looked back and then looked to the future and could see where these people were going and I said, "Hey, I did this internship, I did that internship, if I get a PhD I'll do that or that, and I've already crossed them both off. I've got to find something else to do."

Internships can be extremely important. If your children are interested in an internship in the financial world, I should mention that I used to be a bond trader. I still have lots of friends who now come to me and say "I want your students. I want your students." They hire some of my past students as interns. If they're interested in the financial world, have them send me an email when they're in college and I may be able to hook them up.

Similarly, we may be able to hook them up with -- although the universities can probably do this even better than I can -- what are called Research Experiences for Undergraduates. These are six-, eight-, ten-week programs in the summer where they'll go and work with a top researcher in a field that they're interested in and work with other fascinating students. There's one that is run by (I think it's still the top one) Joe Gallian, who is a past president of the Mathematical Association of America. I'm guessing somewhere between five and ten of the girls that are taking the test in the other building will go to Joe at some point. A lot of the very top students do. Probably five to ten of them will also do the Budapest semesters in mathematics. They go to Budapest and do math there for a semester.

And last, if they need a little walking around money in college, we need as many assistants and graders as we can get our hands on. We have grading teams at all the top
universities around the country. If they need a college job, have them write me. I'm sure I can find something for them to do.

Mostly I've talked about strategies for the students. I want to finish by talking about strategies for the parents. It's very important that you be involved in creating opportunities for your children and for other children.


Thanks to Beth Pauka for providing this transcription of the text of Richard Rusczyk's speech from the video.

