# Math Prize for Girls Olympiad

December 9, 2010

**1.** Let $S$ be a set of 100 integers. Suppose that for all positive integers $x$ and $y$ (possibly equal) such that $x + y$ is in $S$, either $x$ or $y$ (or both) is in $S$. Prove that the sum of the numbers in $S$ is at most 10,000.

**2.** Prove that for every positive integer $n$, there exist integers $a$ and $b$ such that $4a^2 + 9b^2 - 1$ is divisible by $n$.

**3.** Let $p$ and $q$ be integers such that $q$ is nonzero. Prove that

$$\left|\frac{p}{q} - \sqrt{7}\right| \geq \frac{24 - 9\sqrt{7}}{q^2}\,.$$

**4.** Let $S$ be a set of $n$ points in the coordinate plane. Say that a pair of points is *aligned* if the two points have the same $x$-coordinate or $y$-coordinate. Prove that $S$ can be partitioned into disjoint subsets such that (a) each of these subsets is a collinear set of points, and (b) at most $n^{3/2}$ unordered pairs of distinct points in $S$ are aligned but not in the same subset.