1. Let $A_0, A_1, A_2, \ldots, A_n$ be nonnegative numbers such that

$$A_0 \leq A_1 \leq A_2 \leq \cdots \leq A_n.$$ 

Prove that

$$\left| \sum_{i=0}^{\lfloor n/2 \rfloor} A_{2i} - \frac{1}{2} \sum_{i=0}^{n} A_i \right| \leq \frac{A_n}{2}.$$ 

(Note: $\lfloor x \rfloor$ means the greatest integer that is less than or equal to $x$.)

2. Let $\triangle ABC$ be an equilateral triangle. If $0 < r < 1$, let $D_r$ be the point on $AB$ such that $AD_r = r \cdot AB$, let $E_r$ be the point on $BC$ such that $BE_r = r \cdot BC$, and let $P_r$ be the point where $AE_r$ and $CD_r$ intersect. Prove that the set of points $P_r$ (over all $0 < r < 1$) lie on a circle.

3. Let $n$ be a positive integer such that $n + 1$ is divisible by 24. Prove that the sum of all the positive divisors of $n$ is divisible by 24.

4. Let $M$ be a matrix with $r$ rows and $c$ columns. Each entry of $M$ is a nonnegative integer. Let $a$ be the average of all $rc$ entries of $M$. If $r > (10a + 10)^c$, prove that $M$ has two identical rows.