The Advantage Testing Foundation
Math Prize for Girls Olympiad

Thursday, November 15, 2018

Time Limit: 4 hours

1. Let P be a point in the plane. Suppose that P is inside (or on) each of 6 circles $\omega_1, \omega_2, \ldots, \omega_6$ in the plane. Prove that there exist distinct i and j so that the center of circle ω_i is inside (or on) circle ω_j.

2. Let $d(n)$ be the number of positive divisors of a positive integer n. Let $\mathbb{N}$ be the set of all positive integers. Say that a function F from $\mathbb{N}$ to $\mathbb{N}$ is *divisor-respecting* if $d(F(mn)) = d(F(m))d(F(n))$ for all positive integers m and n, and $d(F(n)) \leq d(n)$ for all positive integers n. Find all divisor-respecting functions. Justify your answer.

3. There is a wooden $3 \times 3 \times 3$ cube and 18 rectangular 3×1 paper strips. Each strip has two dotted lines dividing it into three unit squares. The full surface of the cube is covered with the given strips, flat or bent. Each flat strip is on one face of the cube. Each bent strip (bent at one of its dotted lines) is on two adjacent faces of the cube. What is the greatest possible number of bent strips? Justify your answer.

4. For all integers x and y, let $a_{x,y}$ be a real number. Suppose that $a_{0,0} = 0$. Suppose that only a finite number of the $a_{x,y}$ are nonzero. Prove that

$$\sum_{x=-\infty}^{\infty} \sum_{y=-\infty}^{\infty} a_{x,y}(a_{x,2x+y} + a_{x+2y,y}) \leq \sqrt{3} \sum_{x=-\infty}^{\infty} \sum_{y=-\infty}^{\infty} a_{x,y}^2 \,.$$