Math Prize for Girls Olympiad

Advantage Testing Foundation/Jane Street

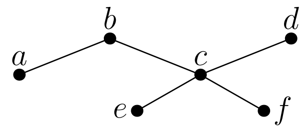
Time Limit: 4 hours

Thursday, December 4, 2025

Problem 1. A positive integer is called *amazing* if you can erase one of its digits (in base 10), optionally reorder the remaining digits, multiply the result by 9, and get the original number. For instance, the number 315 is amazing because $35 \cdot 9 = 315$. Determine whether there are infinitely many amazing numbers not divisible by 10.

Problem 2. Let $n \geq 2$ be an even integer and let T be a tree with n vertices. Show that T has an edge between two vertices whose degrees are either both even or both odd.

(Here, a *tree* is defined as a set of n *vertices* and $n-1$ *edges* between pairs of vertices, such that any two vertices are linked via a path of one or more edges. The *degree* of a vertex is defined as the number of edges using that vertex. For example, if $n = 6$ and T is a tree with 5 edges ab, bc, cd, ce, and cf as illustrated below, then edge bc joins two even-degree vertices: b has degree 2 while c has degree 4.)



Problem 3. Jessica and Hannah play a game. First, Jessica draws a circle Γ and chooses a set S of 13 points on Γ. Hannah sees Jessica's choices and then chooses a set T of 2025 points on Γ. Jessica sees what Hannah chooses.

Then Jessica performs a sequence of operations that modify the set S. In an operation, Jessica chooses two perpendicular chords AB and PQ of Γ, for which A, B, P are all in S, but Q is not. She then removes A, B, and P from S and adds Q into S.

Jessica wins if she reaches a state where S has only one point, and that point is not in T. Otherwise, Hannah wins. Which player has a winning strategy?

Problem 4. Acute triangle PQR is inscribed in triangle ABC, with points P, Q, R lying in the interiors of sides BC, AB, AC, respectively, such that $BP = BQ$ and $CP = CR$. Suppose the tangents to the circumcircle of triangle PQR at Q and R meet at point D. Prove that

$$DB + DC \geq AB + AC.$$