The Seventeenth Annual Math Prize for Girls

Sunday, October 12, 2025

Test Version A

DIRECTIONS

1. Do not open this test until your proctor instructs you to.
2. Fill out the top of your answer sheet. Your student number is on your name tag. Your test version (A, B, C, or D) is above on this page.
3. You may use pencils, pens, and erasers. You may also use the scratch paper that we provide. You may NOT use calculators, rulers, protractors, compasses, graph paper, or books.
4. Write your final answers on the answer sheet. When a problem specifies a particular form for the answer, write your answer in that form. Write your answers clearly. We will collect only the answer sheet from you.

NOTES

1. This test contains 20 problems. You will have 150 minutes (2.5 hours) to take the test. Your score will be the number of correct answers.
2. Figures are not necessarily drawn to scale.
3. Good luck!

1. 100 girls collected 2025 pounds of honey altogether. Each of the girls had an empty barrel. At one point they simultaneously did the following: each of the girls poured all her honey equally to all other girls, using their empty barrels. After that Katrine, one of the girls, had 20 pounds of honey. How much honey (in pounds) did Katrine have before honey redistribution?

Answer: 45

2. How many positive integers less than or equal to 1000 are there that are divisible by 2, 3, or 5, and if they are divisible by any two of the numbers 2, 3, or 5, then they are divisible by all three?

Answer: 501

3. An octagon is inscribed in a circle. It has four consecutive sides of length 10 and four consecutive sides of length $7\sqrt{2}$. What is the radius of the circle?

Answer: 13

4. A regular hexagon has side length $6+2\sqrt{3}$. The hexagon is divided into 6 non-overlapping triangles by connecting each vertex to its opposite vertex. These triangles are then colored alternately black and white, going around the center. An identically colored congruent hexagon is placed over the first covering it precisely, then rotated about its center by $30°$ clockwise. What is the total area of the regions where black overlaps black?

Answer: 54

5. Square $ABCD$ is a sheet of paper. Point P on the interior of side $\overline{AB}$ satisfies $m\angle ADP = 12^\circ$. Point Q is on the interior of $\overline{PB}$. When the paper is folded along a crease that passes through C and Q, vertex B falls upon $\overline{DP}$. What is the degree measure of $\angle QCB$?

Answer: 33

6. For every nonnegative integer n, let a_n be the number of integers m between 1 and 3^{2n}, inclusive, such that either m or $m/3$ is a perfect square. Let $b_n = a_{n+1} - 3a_n$. What is

$$\sum_{k=0}^{\infty} \frac{b_k}{3^k}\ ?$$

Express your answer as a radical in simplest form.

Answer: $\sqrt{3}$

7. Let Q be the set of quartic polynomials with leading coefficient -1, four distinct positive integer roots less than or equal to 100, and whose graph has a vertical line of symmetry. What is the maximum value that any of these quartic polynomials can achieve over the real numbers?

Answer: 1500625

8. Evaluate

$$\sum_{n=0}^{\infty} \frac{\cos(\pi n/6)}{\sqrt{3}^{\,n}}.$$

Express your answer as a fraction in simplest form.

Answer: $\dfrac{3}{2}$

9. A triangle has an inscribed circle of radius 3. One side of the triangle is split into segments of length 1 and 10 by the point where the inscribed circle touches the side. What is the perimeter of the triangle?

Answer: 220

10. You want to make an edge-to-edge tiling of a 10×10 square, without gaps or overlaps, using tiles that are all similar to the 1×2 rectangle (but do not necessarily have integer side lengths). In an “edge-to-edge” tiling, no vertex of one tile lies in the interior of an edge of another. Given that the tile in the lower left corner is a vertical 1×2 rectangle, how many ways are there to do this?

Answer: 104

11. Solve for a:

$$\begin{aligned} a - b + c - d + e &= 1 \\ a - 2b + 4c - 8d + 16e &= 128 \\ a - 3b + 9c - 27d + 81e &= 2187 \\ a - 4b + 16c - 64d + 256e &= 16384 \\ a - 5b + 25c - 125d + 625e &= 78125. \end{aligned}$$

Answer: 16800

12. What is the remainder when you divide $526^{12} - 494^{12}$ by the prime number 1019? (Note that $526 \times 494 \equiv -1 \pmod{1019}$.)

Answer: 532

13. Let $-90 \le A \le 90$ satisfy the equation

$$\sin(A^\circ) = \frac{\dfrac{1}{2} + \displaystyle\sum_{k=0}^{44} \sin(2k^\circ)}{\displaystyle\sum_{k=0}^{44} \sin((2k+1)^\circ)}.$$

What is A?

Answer: 89

14. Let a, b, c, and d be the roots of the quartic polynomial $x^4+x^3+x^2+2$. Determine $|a|^2+|b|^2+|c|^2+|d|^2$.

Answer: 6

15. The vertices of a tetrahedron are located at

$$(0,0,0),\ (10,20,30),\ (20,20,30), \text{ and } (30,10,10).$$

How many lattice points are there either inside the tetrahedron or on its boundary?

Answer: 286

16. Let $S = \displaystyle\sum_{k=0}^{200} (-1)^k \binom{600}{3k}$. What is the prime factorization of S?

Answer: $2 \cdot 3^{299}$

17. Define the sequence y_n recursively as follows: $y_0 = 0$, $y_1 = 451$, and $y_{n+1} = 1560y_n - 901^2 y_{n-1}$, for $n > 0$. Determine the smallest positive integer N such that $y_N < 0$ and $y_{N+1} > 0$.

Answer: 11

18. A rectangular gridwork of roads has 4 horizontal avenues crossed by 5 vertical streets. A vertical or horizontal road that connects adjacent intersections will be called a segment. Let P be the set of paths in the gridwork that begin in the upper left and end in the lower right and that do not go up or to the left. Let S_k be the number of ordered pairs (p, q), where p, $q \in P$, such that p and q share exactly k segments. It happens that $S_1 = 284$, $S_3 = 224$, and $S_4 = 100$. What is S_2?

Answer: 258

19. The sides of $\triangle ABC$ are rational and $AB = AC$. Inside $\triangle ABC$, two congruent circles of radius y are tangent to $\overline{BC}$ and, externally, to each other, with one tangent to $\overline{AB}$ and the other tangent to $\overline{AC}$. A circle of radius x is placed inside $\triangle ABC$ tangent to sides $\overline{AB}$ and $\overline{AC}$ and externally tangent to the circles of radius y. Given that x and y are integers, determine the smallest possible perimeter of the triangle whose vertices are the centers of the three circles.

Answer: 242

20. Define the sequence d_n recursively by $d_1 = 1$ and

$$d_{n+1} = \frac{48\sqrt{n} - 7}{48\sqrt{n+1} + 7} d_n,$$

for positive integers n. Evaluate

$$\sum_{k=1}^{\infty} d_k\,.$$

Express your answer as a fraction in simplest form.

Answer: $\dfrac{55}{14}$