



THE EIGHTEENTH ANNUAL MATH PRIZE FOR GIRLS

Sunday, October 11, 2026

Test Version A

DIRECTIONS

1. Do not open this test until your proctor instructs you to.
2. Fill out the top of your answer sheet. Your student number is on your name tag. Your test version (A, B, C, or D) is above on this page.
3. You may use pencils, pens, and erasers. You may also use the scratch paper that we provide. You may NOT use calculators, rulers, protractors, compasses, graph paper, or books.
4. Write your final answers on the answer sheet. When a problem specifies a particular form for the answer, write your answer in that form. Write your answers clearly. We will collect only the answer sheet from you.

NOTES

1. This test contains 20 problems. You will have 150 minutes (2.5 hours) to take the test. Your score will be the number of correct answers.
2. Figures are not necessarily drawn to scale.
3. Good luck!

1. A line passing through the point $(3, 10)$ intersects the parabola $y = x^2$ in the points (x_1, y_1) and (x_2, y_2) . Determine the minimum possible value of $y_1 + y_2$.

2. We place an infinite series of rectangles R_n , $n \geq 1$, in the coordinate plane defined recursively by the following rules: S_1 is a 1 by 1 square at the bottom left of the first quadrant, R_1 is the bottom half of S_1 , S_{n+1} is the top half of S_n , rotated 90° clockwise about the top half's bottom right corner, and R_{n+1} is the bottom half of S_{n+1} , $n \geq 1$. What fraction of the total area of the rectangles R_n , $n \geq 1$, lies below the line $y = 2x/3$?

3. In the coordinate plane, let $X = (1, 0)$, $Y = (0, 1)$, and $O = (0, 0)$. Let V be the set of points (x, y) such that x and y are integers between 1 and 12, inclusive. Let

$$S = \{\tan m\angle POX + \tan m\angle POY \mid P \in V\}.$$

What is the size of S ?

4. An infinite "T" is formed by attaching the foot of a ray to a point on a line in such a way that the ray and the line are perpendicular. A circle of radius 10 is placed on the plane of the "T" so that the amount of length of the "T" contained within the circle is maximized. Determine the square of the distance of the circle's center to the branch point of the "T".

5. Seven real numbers m_k , $k = 1, \dots, 7$ are chosen uniformly and independently at random from the interval $[-1, 1]$. Determine the probability that the seven lines $y = m_k x + k$, $k = 1, \dots, 7$ exhibit a total of 2 intersections to the right of the y -axis.

6. How many ordered triples of positive integers (a, b, c) are there with $a, b, c \leq 100$ such that $a^2 + b^2 \equiv c^2 \pmod{101}$?

7. The vertices of a kite are labeled A , B , C , and D in clockwise order. The interior angles at B and D measure 160° and 130° , respectively. Let O be the intersection of the diagonals of the kite. The line through B perpendicular to $\overleftrightarrow{CD}$ intersects $\overleftrightarrow{CD}$ at F . Let $G = \overleftrightarrow{AB} \cap \overleftrightarrow{OF}$. Suppose the degree measure of $\angle FGD$ is t° , where $0 \leq t < 180$. What is t ?

8. The points $(0, 56, 80)$, $(0, 72, 80)$, $(0, 51, 75)$, and $(0, 69, 75)$ form the vertices of a trapezoid in xyz -coordinate space. A point light is placed at the coordinates (x, y, z) , with $x > 0$. The shadow cast by the trapezoid onto the xy -plane is a square with sides parallel to or perpendicular to the axes. What is x ?

9. Let $x + yi$ be the root of the polynomial

$$8z^7 + 28z^6 + 56z^5 + 70z^4 + 56z^3 + 28z^2 + 8z + 1$$

where x and y are real numbers, $i = \sqrt{-1}$, and y is maximal. What is y ? Express your answer in the form $a + b\sqrt{c}$, where a and b are rational numbers in simplest form and c is a square-free integer.

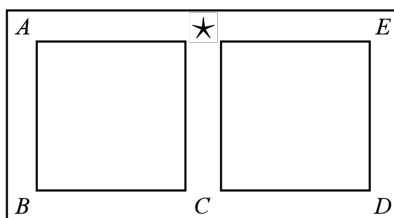
10. Find the smallest positive real number t that maximizes the value of

$$\sin(8t^\circ) - \sin(t^\circ) - \sin(7t^\circ).$$

11. Find the positive integer n that satisfies:

$$\binom{n}{4} = \binom{132}{4} + \binom{190}{4}.$$

12. In the street map below, Ally is located at corner A and Bea is located at corner B .



Every 3 minutes, each picks one of the cardinal directions she could go from her current position, uniformly and independently at random, walks in that direction to the first corner or intersection encountered, and stays there until the next direction selection. What is the probability that Ally reaches intersection $\star$ before Bea?

13. Let c be a positive real number. Place 2,025 red circles of radius $1/c$ on the coordinate plane, one centered at each ordered pair of integers (x, y) with $-22 \leq x, y \leq 22$. Find the largest c for which every line passing through the origin has a common point with at least two red circles. Express your answer as $\sqrt{d}$ where d is a square-free integer.

14. Let F_k be the Fibonacci sequence, defined recursively by $F_1 = F_2 = 1$ and $F_{n+1} = F_n + F_{n-1}$ for $n > 1$. Evaluate

$$\sum_{n=1}^{\infty} \frac{1}{F_n F_{n+1} F_{n+2} F_{n+4} F_{n+5} F_{n+6}}.$$

(Note: The denominator does *not* contain a factor of F_{n+3} .)

15. Consider the “circle” of quadratic polynomials $x^2 - bx + c$ where

$$(b + 18)^2 + (c + 15)^2 \leq 512.$$

What is the largest root of a quadratic in this collection?

16. Let

$$s = \sum_{n=1}^{\infty} \arctan \frac{8n}{16n^4 + 5}.$$

Determine $\tan s$.

17. Consider a regular icosahedron inscribed in a sphere S of radius 1. For each vertex V of the icosahedron, let $C(V)$ be the great circle of S that has a pole at V . These great circles partition S into spherical polygons. Let A be the area of one of the spherical pentagons in this pattern. Determine $\cos(A)$. Express your answer in the form $\frac{a\sqrt{d}}{b}$, where a and b are relatively prime positive integers and d is a square-free integer.

18. Let

$$M = \begin{pmatrix} 51 & -46 & -22 \\ 34 & -31 & -14 \\ 38 & -34 & -17 \end{pmatrix}.$$

Let $v = M^{1000} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$. Determine the first (top) component of v .

19. Iris arranged 4 apples, 4 bananas, and 4 caramels into three distinct rows by type, to be eaten one at a time as snacks. Snacks of the same type are indistinguishable. To pick a snack, she must obey the following rules:

- Take a snack only from the right end of a row.
- Never choose consecutive snacks of the same type.

How many different ways are there for her to have eaten all 12 snacks?

20. Jackie plays a game in a series of turns. She starts the game with 1 unit each of two resources A and B. For each turn, she can choose to either:

- Gain 0.25 B resources immediately for each A resource she has.
- Gain 1 A resource immediately and 1 A resource right after her next turn, for each B resource she has.

Let a_n be the maximum number of A resources Jackie can have after n turns. The value of $\lim_{n \rightarrow \infty} \sqrt[n]{a_n}$ can be expressed in the form $\sqrt[m]{x + y\sqrt{z}}$ where x and y are rational, and z and m are positive integers with z square-free. Determine the smallest possible value of $m + z$.

